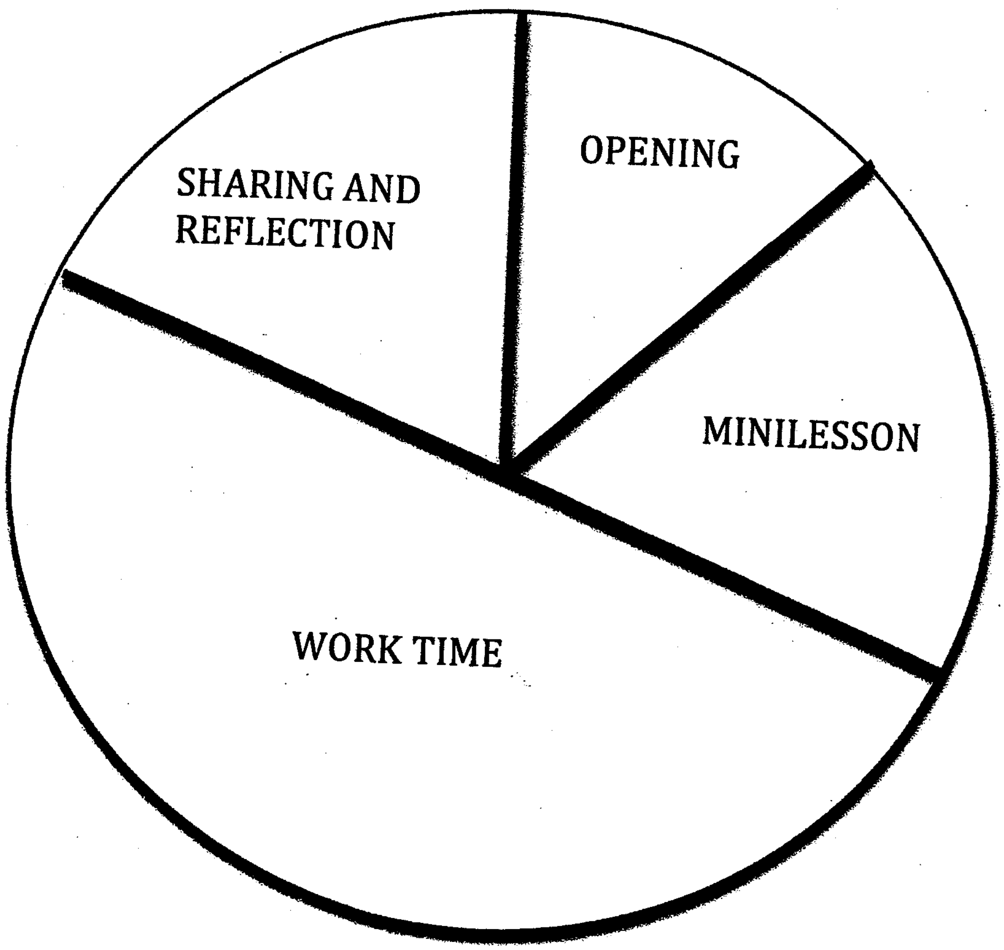
# Math Workshop Model



## Mathematics | Standards for Mathematical Practice Common Core State Standards for Mathematics

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 . 8 equals the well remembered 7 . 5 + 7 . 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 . 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

## 8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## 1. Lesson Opening

**Goals:** The goal is to engage the whole class in mathematical thinking, to activate background knowledge and to encourage student discourse; the opener sends the learners the important message: you know what to do, you are here to work, and you are capable of individual success as a mathematician. When the teacher starts class by inviting and encouraging student thinking, one conveys the important message that math class is about students working to hone ideas and solve problems.

**Description:** The "Opening" should be a task that is accessible to all students with multiple entry points and the flexibility to be extended. The teacher is the facilitator and the students are actively listening and engaged in thinking about the task presented.

**Examples:**

- A Daily Routine:
  - **Grades K-2:** Number Talk, Number Strings, Start With Get To, Ten Wand, Count Around the Circle, Quick Images, Target Number, What Do You Know, Build It, Today's Number, True/False Equations, What Time Is It
  - **Grades 3-5:** Number Talks, Today's Number, Number Puzzles, Count Around, Guess My Rule, Number Line, Sparkle, Talk a Mile a Minute, Eliminate It, Guess My Rule, Over Under, Broken Calculator, Line Them Up, Calculator Digit Change, Decimal Between, Who Am I, Frayer Model
  - For directions to the above routines and downloads enter the following website from Howard Public School System with your grade level. For example: **www.grade4commoncoremath.wikispaces.hcpss.org**

- Group Problem Solving Task / Word Problems
- Focus on a Mathematical Statement (about behaviors of mathematicians) or a Standard for Mathematical Practice
- Math Vocabulary Development
- Entry Slip
- A Homework Problem
- Conceptual Conversations / Previous Learning Connections
- Quick Write
- Tally Check: This is a simple system for checking homework. Write a list of the problem numbers on the board. In the first few minutes of class, invite students to make a tally mark next to any of the problems they would like help with. Choose 2-3 problems with which most students struggled and use them to engage students in a group discussion. (Tracey Shaw, HS teacher)
- Share and Compare
  Put students into small groups to look over their homework and then discuss one or more discrepant answers. Require students to justify their thinking and defend their solutions with the goal being to understand how to complete all problems correctly.

See Anchor Chart 1

| Teacher | Student |
| --- | --- |
| - Sending a clear message: you know what to do, you are here to work, and you are capable of mathematical success<br>- Telling stories<br>- Posing problems<br>- Providing tools<br>- Making observations<br>- Allowing collaboration<br>- Asking guiding questions<br>- Facilitating routines, hooks, and/or discussions | - Processing information<br>- Listening<br>- Discussing problems<br>- Using math tools<br>- Modeling thinking |

**2. Math Mini Focus Lesson**

**Goals:** The goal is to identify and set the mathematical purpose of the day, mentor/model the mathematical "thinking" necessary for students to achieve the purpose.

**Description:** The lesson is tied to a specific math standard and launches students into an exploration of a problem. The teacher might model thinking strategies and reasoning through content that is grounded in mathematical practices. Checking for student understanding before transitioning to "exploration or work time".

**Mini Lesson on Problem Solving**
Reading a book that focuses on the content of the lesson
Asking a focusing question(s)
Posing a simpler problem that connects to your focus task
Building content understanding
Offering real-life examples
Sketching anchor charts
Explaining in the context of math
Thinking aloud
Showing ways to hold thinking
Using parallel problems
Addressing misconceptions
Introducing a rich problem or game
Building upon a variety of representations/models/tools

See Anchor Charts 1, 2, 3, 4, and 5

| Teacher | Student |
|---|---|
| - Asking for prior knowledge, reviewing past learning<br>- Introducing rigorous tasks that drive the understanding of focused concepts, procedures or skills<br>- Explicit modeling ("think aloud")<br>- Questioning<br>- Asking students to model<br>- Clarify misconceptions<br>- Modeling Mathematical Practices<br>- Explicitly describing what you want the students to do and what quality is<br>- Developing vocabulary | - Explaining why<br>- Asking questions<br>- Modeling with tools<br>- Being active decision makers<br>- Answering question<br>- Using vocabulary<br>- Developing and practicing the Mathematical Practices<br>- Developing basic fact procedures and skills |

### 3. Work Time: Exploration and Using the Practices to Make Meaning

**Goals:** The goal is to train students to engage in the Mathematical Practices during an exploration of math content and/or a problem-solving task.

**Description:** Students learn most when they spend math work time thinking, talking, and making meaning of mathematics for themselves. Half of the total math workshop time should be devoted to this work time for students to work individually, with a partner, or in small groups. This time may look different depending on the needs and cognitive demands of the task. Providing differentiation in content, process, or product is a way to help all students persevere to complete assigned task(s). The teacher circulates among the students, asking probing questions, collecting formative assessment data, extending learning opportunities, looking for misconceptions, catching and releasing, and clarifying and nudging students to become problem solvers.

**Examples:**
Routine and Non-routine Word problems
Games
Purposeful Centers – aligned to standard(s) being focused on
Independent, partner, group work
Choice
Rich problems
Performance Tasks
Post Assessment
Math Exchanges Groupings/Invitational Groups

See Anchor Charts 2, 6, and 7
See Anchor Chart 8 Teacher Moves to Support Student Dialogue: Sharing of Strategies and Focus on Identifying Efficiency
Anchor Chart 11 Table 1 and 2: Common Word Problem Types

| Teacher | Student |
|---|---|
| - Catching and releasing<br>- Providing tools<br>- Circulating and asking probing questions<br>- Scaffolding as needed – differentiating content, product, process<br>- Extending thinking<br>- Using informal observations, note-taking<br>- Using formative assessment<br>- Monitoring student understanding | - Working independently, with a partner, in small groups of 2 to 4 collaboratively<br>- Planning and problem-solving<br>- Looking for and investigating patterns<br>- Representing<br>- Recording<br>- Showing evidence of learning<br>- Communicating ideas to others<br>- Listening to others ideas<br>- Playing a meaningful game<br>- Applying past learning to new situations<br>- Practicing basic facts for mastery |

### 4. Sharing and Reflection

**Goals:** This is an essential time to connect the math work to the purpose of the day's lesson, share strategies for problem solving, thinking, and questions, respond to the thinking of others and to consider how other students' ideas have changed and grown.

**Description:** Students and teacher gather together as a large group to discuss their learning, strategies, and reasoning. Students may look for patterns in the work shared, and teachers can address misconceptions that arise, as well as plan next steps based on what was observed in student work and during student discussions.

See Anchor Charts 3, 4, 5, 9, and 10

| Teacher | Student |
|---|---|
| - Facilitating discussions<br>- Clarifying misconceptions<br>- Summarizing key learning<br>- Asking probing questions that lead to deeper understanding<br>- Collecting data on student needs based on responses | - Sharing strategies, solutions, thinking<br>- Discussing the ideas of others<br>- Critiquing the reasoning of others<br>- Finding patterns, making generalizations |

Anchor Chart 1

Planning an Opening Problem

| | About a Problem | About a Concept |
|---|---|---|
| Background knowledge | What does this remind you of? | What do you already know about pyramids? |
| Asking questions | What are all the questions you can ask about this problem? | What questions do you have about pyramids? |
| Determining importance | What would be important to think about if you were going to solve this problem? What data are significant? Where would you begin? | What is important to remember about surface area? |
| Inferring | What might be some stumbling blocks to solving this problem? | How is the concept of surface area used in daily life? |
| Mental models | Represent this situation in two or more ways. | Create a model to represent what you know about pyramids. |
| Monitoring for meaning | (Given a solution to a problem) Does this answer make sense? Why or why not? Explain. | What do you understand now about pyramids? What is confusing about them? |
| Synthesis | How does this relate to other things we know? | What do triangles have to do with pyramids? |

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Anchor Chart 2

Asking Learners to Share Their Problem-Solving Processes

For learners to explain their own solutions, we need to physically and verbally get out of the way; move away from the front of the room, listen more than you talk, interject only as needed to prompt further verbalization, let the student presenter have the floor, and be patient.

Encourage a student (speaking either to you individually, to a partner or small group, or to the whole class) by inviting them to:

| Sharing and Reflection | Question |
|---|---|
| Describe their process | How did you start? Talk though your thinking? |
| Reflect on their decisions | What were some of the decisions you made as a problem solver? |
| Explain their vigilance | What were some of the speed bumps you encountered when solving this problem? |
| Confirm their thinking | How do you know you are right? |
| Make connections | What is the big idea of this problem, and how could you apply this concept to other problems? |
| Promote discourse | What kind of feedback would you like from me/the group? |

Anchor Chart 3

Typical Sharing Vs. Minds-On Discourse

| | Typical Sharing | Minds-on discourse |
|---|---|---|
| Positions | Teacher remains at the front of room. Student presenter may share from desk, give teacher his/her paper, or come to front. | Student presenter stands at front of group while teacher steps to the side. |
| Content | Student shows or shares an answer. | Student explains how he/she approached the problem and why he /she believes his/her solution is accurate. |
| Response | Teacher verifies accuracy of student's answer or corrects any errors. | Classmates respond with questions and feedback, compare their ideas to those of the presenter, share alternate approaches. |

In responding to the ideas of others, students could start with:

- I agree with . . . because . . .
- I disagree with . . . because . . .
- I am wondering . . .
- How did you know to . . .
- Can you explain . . .

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Anchor Chart 4

Taking Time to Uncover Errors

Frame mistakes as opportunities to investigate further; compliment those willing to share their thinking, whether correct or otherwise.

Encourage this reflection by inviting learners to:

| Reflect | Question |
|---|---|
| Share multiple solutions | What other answers did folks get? |
| Pinpoint decisions | What were the critical decision points in solving this problem? |
| Assess errors' impact | How would a wrong turn affect your world? |
| Categorize mistakes | What kinds of errors made you stumble? |
| Identify signposts | Were there any clues that you might have missed along the way? |
| Generalize | What do we need to remember in order to solve problems like these accurately? |

Anchor Chart 5

Conversation Formats for Sharing Mathematical Thinking

| Structure | Description | Sentence Starters |
|---|---|---|
| Pair and share | Students turn to a neighbor and briefly share their own thinking, then listen to the thinking of their partner. | "I am thinking . . .because . . ."<br><br>"I had a different idea. I was thinking . . . because . . ." |
| Problem discussion | In small groups, students work together to solve a problem. | "I think we should . . . because . . ."<br><br>"I disagree with . . . because . . ."<br><br>"What if we . . . " |
| Carousel discussion | In small groups, students either respond to written information or gather thinking on a topic by writing on a shared piece of paper; the paper is then passed on to the next group. | "What do you think about . . ."<br>"I am wondering . . . "<br>"I don't understand . . ."<br>"One question I have is . . . " |
| Peer critique | One student comes to the board and presents his/her work or thinking about a certain problem to the whole class; peers observe and then respond with comments, questions, and comparisons. | After the presenter explains, peers may say,<br>" I like how you . . ."<br>"Can you help me understand . . . "<br>"Why did you . . ." |

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Anchor Chart 6

Strategies for Differentiation

| | **Content** | **Product** | **Process** |
|---|---|---|---|
| Strategies for differentiation | Changing the numbers in a problem<br><br>Adjusting the reading level of the text<br><br>Adding layers or steps to finding the solution<br><br>Giving different amounts of work to different groups | Flexibility with how learners demonstrate understanding<br><br>Choice in final format<br><br>Clear descriptors of quality<br><br>Intentional scaffolding | Breaking tasks into small steps<br><br>Choice in whether to partner and with whom<br><br>Choice of resource materials<br><br>Choice of problem solving strategies<br><br>Choice of mental models |

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**Anchor Chart 7**

**Instructional Routine: Exploration and Making Meaning**

**The Work of Work Time**

| Students: What to do | | Teacher: What to do |
|---|---|---|
| Generate questions about the topic of the upcoming unit | | Being clear about what you are asking for is worth the time invested. Sharing a page of student work and asking students, "What do you notice about this learner's work?" This conversation can lead to what finished work should look like: legible, neat, labeled, accurate, with units, and so forth. |
| Wrestle to solve one juicy problem in a table group, record their thinking, then work independently on some practice problems | | Be sure that students understand there is no "done". They should understand that they have additional responsibility when they complete the assigned work, if time allows, such as additional practice, supporting others, reading about math, practicing basic skills games. |
| Work independently to complete a problem or problem set from the text, document their thinking, then shift to sharing and discussing their work with a small group | | Be clear about what the class is doing and about the how and the why. Effectively establish and convey the purpose from your mini-lesson and explain the connection between purpose and the task during the transition to work time, so that your students remain conscious of why they are doing what they have been asked to do. |
| Work with a partner to create a concept map that clarifies the relationship between important unit vocabulary terms. (See example on next page) | | Confer with small groups or individuals to promote thinking, gather data, and troubleshoot when necessary. Resist the temptation to police and cajole. |
| | | Engage in conversations with learners about their thinking and be a facilitator of mathematical insight. |
| | | Catch and Release - a quick break in the action where a teacher gathers everyone's attention to deliver some information. |



**(Anchor Chart 7 continued)**

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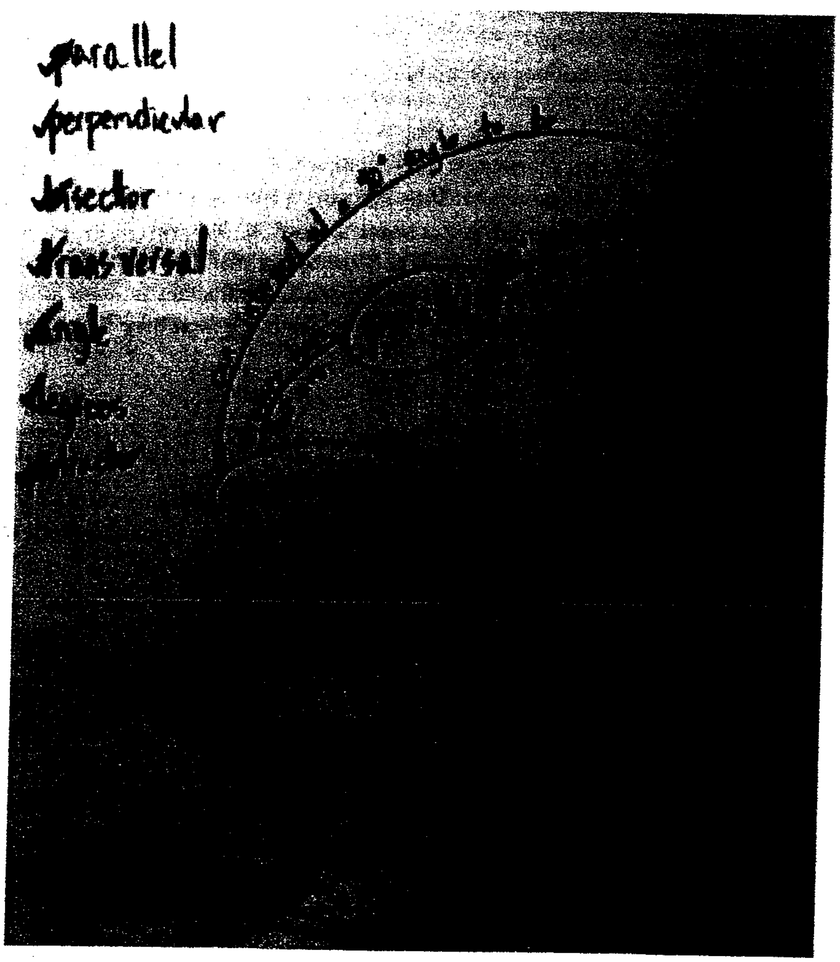
**What is a Concept Map?**
Synthesis is the act of growing your thinking, noticing how your new learning connects to prior knowledge. Concept mapping is one way for learners to gather and synthesize what they know about a given topic. It is appropriate before, during, and after a learning activity – as a pre-assessment or a review of a unit.

1. Create a list of important words or ideas to be synthesized.
2. Each word or idea is placed in a circle and a line is drawn to connect it to related words or ideas.
3. Students explain the connection in their own words in as much detail as possible before drawing the next line to avoid ending up with a wordless sketch of a spider.
4. While mapping, learners may elect to add additional words or concepts in circles of their own to help make the connections stronger.

Example:



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Anchor Chart 8

**Figure 5.4** Teacher Moves to Support Student Dialogue: Sharing of Strategies

| Teacher Language | Purpose | Teacher Look-Fors and Follow-Ups |
|---|---|---|
| **Initiating Dialogue** | | |
| *Who has a strategy for figuring out this problem?* | Opening the sharing to all. | Are the same kids always volunteering to share? |
| *Ana, I saw you working on an interesting strategy. Can you share what you have been thinking with us?* Or, *Ana, I think your strategy might help other people think about this problem. Will you share it when we are ready to reflect?* (I usually check with this person while the students are working to make sure he or she is comfortable sharing.) | Focusing the opening of the sharing on one person. This could be a person who is sometimes reluctant to share, a person who thought about the problem in an efficient way, or a person whose strategy is similar to or different than others' strategies. | How many have a strategy they are willing to share?<br>Are kids with more and less sophisticated strategies volunteering to share? |
| *We are going to focus on three people's strategies today. As they share, think about how your strategy was similar to or different from those that Ana, Shereen, and Rafa used.* | Sharing three strategies that are representative of others' thinking for a comparative conversation of strategies. | Which students recognize their own strategies or ideas in the work of others? |
| *Turn and talk to the person next to you. Share your strategies. Work really hard to understand your partner's strategy by looking at his or her work and asking questions.* Signal for kids to turn back into the group. (I use a shaker.) *Who is ready to share their partner's strategy?* | Giving all students an opportunity to share their strategies and practice meaningful dialogue. As a teacher, I can listen in on particular conversations to assess strategies and dialogue skills. | Who is actively listening and talking with his or her partner about his or her strategy/work?<br>Who can understand his or her partner's strategy well enough to explain it? |
| **Follow-Up After a Student Has Shared a Strategy** | | |
| *Shereen shared some important ideas. Who can say how Shereen solved this problem?* | Learning to restate the ideas and add on to the ideas of others. | Are children simply "parroting" Shereen's strategy word by word or are they restating a strategy in a way that demonstrates understanding? |
| *Who can add on to what Shereen shared?* | Encouraging children to listen to each other's ideas with a specific purpose in mind (rephrasing a strategy, adding on a new idea, comparing strategies). | Who can follow someone else's line of thinking and add his or her own response or idea that relates to the initial idea?<br>Who shared an unrelated idea because he or she either lacks understanding/misunderstands Shereen's strategy or because he or she doesn't understand that his or her thought should be related to that of Shereen? |
| *Why do you think Shereen chose to ...?* (Point out something important/different/efficient in the student's strategy.) | Learning to engage in dialogue that encourages thinking about their own and others' ideas. | Who understands the reason Shereen chose her strategy for this specific problem?<br>Who simply restates Shereen's idea without addressing her reasons behind using this strategy? |

*(continued)*

## Anchor Chart 8

**Figure 5.4** Teacher Moves to Support Student Dialogue: Sharing of Strategies *(continued)*

| | | |
|---|---|---|
| *What do you all think? Do you agree or disagree with Shereen's answer?* | Demonstrating a high level of thinking by understanding and articulating someone's motivation for using a specific strategy. For example, "Shereen made the 79 into an 80 when she was adding 79 plus 14 because it's an easier number to add 14 to. Then when she got 94, she had to take out the 1 she added before to the 79. So she got 93." | Are students actively engaged in listening and evaluating the strategy for correctness and efficiency?<br>Do children understand where someone got off track, which resulted in a wrong answer? |
| *Did anyone solve this problem in a similar/different way than Shereen?* | | Who can distinguish whether his or her strategies and ideas are similar to or different from those of Shereen?<br>Who can articulate *why* they are different? |
| **Eliciting More Information from a Student Who Has Shared an Idea** | | |
| **Teacher Language** | **Purpose** | **Teacher Look-Fors and Follow-Ups** |
| *Tell us more. Or, Tell what you mean by . . .* | Giving student opportunity to expand if his or her idea is unclear or you want to further highlight this idea. | Does the student know how to clarify ideas? |
| *Hmm . . . I'm not sure I understand exactly what you mean. Sometimes kids understand each other's strategies better than adults. Does anyone understand what Rafa is telling us?* | It's true! Sometimes I can be at a complete loss for what a student is saying, but other kids are nodding and connecting to the speaker's idea. Also, this reinforces to the speaker and listeners that ideas must be presented clearly and listeners must work to understand. | Is there negotiation of meaning between the original speaker and those clarifying the idea? |
| *Keep going. You have something important to tell us.* | Responding to a student stopping mid-idea or saying, "I forgot." Often a few words from the teacher will give confidence to keep going with the idea. | Do students become more confident in presenting ideas, or are they truly confused? |
| *Can you show us how you did that?* | Sometimes ideas can be hard to explain orally and are more clearly illustrated/recorded/shown by the person who thought of them. | How does the student demonstrate what he or she did? With math tools? A written strategy? A drawing? |
| *I think it might help to see your work. Is it okay if I hold your paper up while you explain what you did?* | Many times students explain their ideas from their written work, which may be hard for others to follow. Sometimes it helps if the teacher holds the work while the student shares so listeners can refer to the students' words and work. | |

Anchor Chart 8

**Figure 5.5** Teacher Moves to Support Student Dialogue: Focus on Identifying Efficiency

| Teacher Language | Purpose | Teacher Look-Fors and Follow-Ups |
|---|---|---|
| *Remember how mathematicians look for efficient strategies to solve problems? I noticed some efficient strategies when you all were solving this problem. Max, can you share your strategy with us?* | Reminding children that efficiency is one of our goals in math, acknowledging that many of them were using efficient strategies, and focusing on a child whose strategy I want to highlight for efficiency. | Who recognizes that his or her strategy was similar to or different from Max's?<br>Who can compare his or her strategy to Max's?<br>Who can explain part or all of what Max did and why he did what he did? |
| *Who understands some of what Max told us?* Or, *Eva, what part of Max's strategy do you understand?* | Giving other children the opportunity to practice explaining an efficient strategy. By using the words *some of* or *part of*, I let them know it is okay not to fully understand the strategy. This is an effective way to get kids to start talking. Often they understand more than they think. | What parts of Max's strategy are highlighted?<br>Can children identify why Max's strategy was efficient? |
| *So, Max told us that he added six to fifty-two. Who can show us what that looks like and sounds like?* Or, *So, Max started counting from fifty-two. Is it okay to start with the second number in the addition number sentence?* | Focusing on the specific efficient strategy (counting on from larger number). Allowing several children to practice it. "Fifty-two . . . 53, 54, 55, 56, 57, 58."<br><br>Focusing on a specific big idea (commutative property of addition). Students must prove why or how this works. | |

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**Anchor Chart 9**

**Instructional Routine: Sharing and Reflection**

**Presenting Ideas to the Whole Group**

| | Typical "Correcting" | Minds on Sharing |
|---|---|---|
| Focus on | Answers | The thinking behind the solutions |
| Students' Role | Figuring out if they got it "right" | Understanding the ideas of peers<br>Reconsidering their own approach and solution |
| Teachers' Role | Assessing which is correct<br>Correcting mistakes | Probing for deeper explanations<br>Inviting thinking by listeners<br>Gathering data on students' needs |
| Role of Errors | Correcting by telling right answers | Exploring to uncover and address the thinking that led that problem solver astray |



*Minds on Mathematics Using Math Workshop to Develop Deep Understanding in Grades 4-8* by Wendy Ward Hoffer, p.157 Heinemann 2012

Anchor Chart 10

Sharing and Reflection

| | Prompts |
|---|---|
| 1 | How has your thinking solidified or changed? |
| 2 | What patterns have you noticed? |
| 3 | Share one thing you learned from another mathematician. |
| 4 | What was your biggest "aha" moment today? |
| 5 | What questions do you have about what you learned today? |
| 6 | What is the most important thing you want to remember? |
| 7 | Write about something you wish we had more time to investigate. |
| 8 | Tell of one thing you understand; one you are still confused about. |
| 9 | Give feedback for the teacher about how today's class could have been even better. |
| 10 | What will you say when your friends ask you, "What was math class about today?" |
| 11 | If we were starting class with a pop quiz tomorrow, what would the questions be? |
| 12 | What do you think we are going to do next? |
| 13 | How did you use your background knowledge today? |
| 14 | What was important about today's lesson? |
| 15 | I used to think ________. Now, I think ________. |
| 16 | What? So what? Now what? |

## Anchor Chart 11

TABLE 1. Common addition and subtraction situations.[6]

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| Add to | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?<br>2 + 3 = ? | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?<br>2 + ? = 5 | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?<br>? + 3 = 5 |
| Take from | Five apples were on the table. I ate two apples. How many apples are on the table now?<br>5 - 2 = ? | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?<br>5 - ? = 3 | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?<br>? - 2 = 3 |

| | Total Unknown | Addend Unknown | Both Addends Unknown[1] |
|---|---|---|---|
| Put Together/ Take Apart[2] | Three red apples and two green apples are on the table. How many apples are on the table?<br>3 + 2 = ? | Five apples are on the table. Three are red and the rest are green. How many apples are green?<br>3 + ? = 5, 5 - 3 = ? | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?<br>5 = 0 + 5, 5 = 5 + 0<br>5 = 1 + 4, 5 = 4 + 1<br>5 = 2 + 3, 5 = 3 + 2 |

| | Difference Unknown | Bigger Unknown | Smaller Unknown |
|---|---|---|---|
| Compare[3] | ("How many more?" version):<br>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?<br><br>("How many fewer?" version):<br>Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?<br>2 + ? = 5, 5 - 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?<br><br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have?<br>2 + 3 = ?, 3 + 2 = ? | (Version with "more"):<br>Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?<br><br>(Version with "fewer"):<br>Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?<br>5 - 3 = ?, ? + 3 = 5 |

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of National Research Council (2009, op. cit., pp. 32, 33).

## Anchor Chart 11

TABLE 2. Common multiplication and division situations.[7]

| | Unknown Product<br>3 × 6 = ? | Group Size Unknown<br>("How many in each group?" Division)<br>3 × ? = 18, and 18 ÷ 3 = ? | Number of Groups Unknown<br>("How many groups?" Division)<br>? × 6 = 18, and 18 ÷ 6 = ? |
|---|---|---|---|
| Equal Groups | There are 3 bags with 6 plums in each bag. How many plums are there in all?<br>*Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether? | If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?<br>*Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be? | If 18 plums are to be packed 6 to a bag, then how many bags are needed?<br>*Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have? |
| Arrays,[4] Area[5] | There are 3 rows of apples with 6 apples in each row. How many apples are there?<br>*Area example.* What is the area of a 3 cm by 6 cm rectangle? | If 18 apples are arranged into 3 equal rows, how many apples will be in each row?<br>*Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? | If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?<br>*Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? |
| Compare | A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?<br>*Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? | A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?<br>*Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? | A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?<br>*Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first? |
| General | $a \times b = ?$ | $a \times ? = p$, and $p \div a = ?$ | $? \times b = p$, and $p \div b = ?$ |

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.